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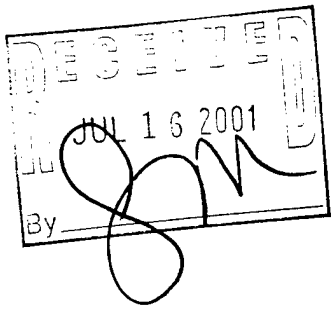
13. ABSTRACT (Maximum 200 words)

We have found the best solution to Duncan-Mortensen-Zakai equation for Kalman -Bucy filtering system with arbitrary initial condition. We show that this equation can be solved explicitly with an arbitrary initial condition by solving a system of ordinary differential equations and a Kolmogorov type equation. The Kolmogorov equation can be computed off time. We have given several algorithms to do parallel computation for the system of ordinary differential equations. As a result, we achieve the real-time solution to Duncan-Mortensen-Zakai equation for Yau filtering system with initial condition. Under Yau's direction, Professor Yeu-Tai Lai and his students in the Department of Electrical Engineering, National Cheng Kung University, Taiwan, has successfully implemented an ODEs solver for the Yau filtering system in hardware.



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Sincerely,

Stephen Yau, Ph.D.

Simulation and Computation Research on Nonlinear Filtering Final Report

Stephen S.-T. Yau

March 31, 2001

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A. Statement of problem studies

The filtering problem considered here is based on the following signal observation model:

$$\begin{cases} dx(t) = f(x(t))dt + g(x(t))dv(t) & x(0) = x_0 \\ dy(t) = h(x(t))dt + dw(t) & y(0) = 0 \end{cases} \tag{1}$$

in which x, v, y and w, are, respectively, $\mathbf{R}^n$, $\mathbf{R}^p$, $\mathbf{R}^m$ and $\mathbf{R}^m$ valued processes, and v and w have components which are independent, standard Brownian process. We further assume that $n = p$, f, h are C^∞ smooth, and that g is an orthogonal matrix. We will refer to $x(t)$ as the state of the system at time t and $y(t)$ as the observation at time t.

Let $\rho(t,x)$ denote the conditional probability density of the state given the observation $\{y(s); 0 \leq s \leq t\}$. It is well-known that $\rho(t,x)$ is given by normalizing a function, $\sigma(t,x)$, which satisfies the following Duncan-Mortensen-Zakai equation:

$$d\sigma(t,x) = L_0\sigma(t,x)dt + \sum_{i=1}^{m} L_i\sigma(t,x)dy(t), \sigma(0,x) = \sigma_0 \tag{2}$$

where

$$L_0 = \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2}{\partial x_1^2} - \sum_{i=1}^{n} f_i\frac{\partial}{\partial x_i} - \sum_{i=1}^{n}\frac{\partial f_i}{\partial x_i} - \frac{1}{2}\sum_{i=1}^{m} h_i^2$$

and for $i = 1, \ldots, m$, L_i is the zero degree differential operator of multiplication by h_i. σ_0 is the probability density of the initial point, x_0.

Equation (2) is a stochastic partial differential equation. In real applications, we are interested in constructing robust state estimators from observed sample paths with some property of robustness. Davis studied this problem and proposed some robust algorithms. In our case, his basic idea reduces to refining a new unnormalized density

$$u(t,x) = \exp\left(-\sum_{i=1}^{m} h_i(x)y_i(t)\right)\sigma(t,x).$$

It is easy to show that $u(t,x)$ satisfies the following time varying partial differential equation

$$\begin{aligned} \frac{du}{dt}(t,x) &= L_0u(t,x) + \sum_{i=1}^{m} y_i(t)[L_0, L_i]u(t,x) \\ &\quad + \frac{1}{2}\sum_{i,j=1}^{m} y_i(t)y_j(t)[[L_0, L_i], L_j]u(t,x) \\ u(0,x) &= \sigma_0 \end{aligned} \tag{3}$$

where $[L_0, L_i]$ denotes the Lie bracket of L_0 and L_i.

The problem is to solve explicitly equation (3) in real-time.

B. Summary of the most important results

(I) In [Mi1], Mitter pointed out that the innovation approach to nonlinear filtering theory is not, in general, explicitly computable. It was first proposed by Brockett and Clark [Br-Cl], Brockett [Br], and Mitter [Mi1] to use estimation algebras to construct finite-dimensional filters. The idea is to use the Lie algebraic method to solve the Duncan-Mortensen-Zakai (DMZ) equation, which is a stochastic partial differential equation. By working on the robust form of the DMZ equation, we can reduce the complexity of the problem to that of solving a time-variant partial differential equation.

In the past decade, the Lie algebriac viewpoint has been remarkably successful, and recent works[T-W-Y], [D-T-W-Y], [Ya], [Ch-Ya], [Ch-Ya1], [Ch-Ya2], [C-L-Y1], [C-L-Y2], [Hu-Ya2], [Hu-Ya3], [Wo-Ya] have given us a deeper understanding of the DMZ equation, which was essential for progress in nonlinear filtering as well as in stochastic control. In fact it was Yau (cf. [Ya1], [Ya2]) who first used Lie algebraic method to discover the most general class of finite dimensional filters which include linear filters and exact filters as special cases. In [Ch], this general class of finite dimensional filters are called Yau filters.

In spite of the success of the Lie algebra method, it is extremely desirable to treat the DMZ equation by a direct method.

Despite its usefulness, the Kalman-Bucy filter is not perfect. One of its weaknesses is that it needs a Gaussian assumption on the initial data. The situation is more complex when the statistics of the initial condition are modeled by an arbitrary distribution. In the case where the linear filtering system (i.e., f, g, and h are linear functions in (1) is completely reachable and completely observable, Hazewinkel observed on p. 115 of [Ha] that the estimation algebra E (i.e., a Lie algebra generated by differential operators L_0, $h_1(x), \ldots, h_m(x)$) is the $2n+2$ dimensional Lie algebra with basis L_0, $\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}$, $x_1, \ldots, x_n$, 1. Even in this case, the Wei-Norman approach used to find the solution of (3) is more complicated than the procedure in Theorem 1 below because not only must one solve a finite system of ordinary differential equation and a Kolmogorov equation, but also one has to integrate n partial differential equations corresponding to the operators $\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}$. More important, if the linear system is not completely reachable or completely observable, then the basis of the estimation algebra is not explicitly known (although it can be computed). As a result, there is an additional disadvantage of the Wei-Norman approach, namely, one cannot write down the finite system of ordinary differential equation explicitly in the non-maximal rank case. The novelty of Theorem 1 is that our finite system of ordinary differential equations is explicitly written down and our procedure to get the solution of (3) is simpler than the Lie algebra approach. Most importantly, our theorem works for Yau filtering system which includes linear filtering system and exact filtering system as special cases.

Theorem 1 *Consider the Yau filtering system* (1) *with arbitrary initial condition,* i.e.,

(Y1) $f_i(x) = \ell_i(x) + \dfrac{\partial F}{\partial x_i}(x)$, $1 \le i \le n$, *where* $\ell_i = \displaystyle\sum_{j=1}^{n} d_{ij}x_j + d_i$ *for* $1 \le i \le n$ *and* F *is a* C^∞ *function.*

(Y2) $h_i(x) = \sum_{j=1}^{n} c_{ij}x_j + c_i,\ 1 \le i \le m.$

(Y3) $\eta(x) := \sum_{i=1}^{n} f_i^2(x) + \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i} + \sum_{i=1}^{m} h_i^2(x) = \sum_{i,j=1}^{n} \eta_{ij}x_ix_j + \sum_{i=1}^{n} \eta_i x_i + \eta_0$, *where* η_{ij}, η_i *and* η_0 *are constants.*

Choose a C^∞ *function* $G(x)$ *such that*

$$\Delta G(x) + |\nabla G|^2(x) + 2\sum_{i=1}^{n} \ell_i(x)\frac{\partial G}{\partial x_i}(x) = \eta(x) - \sum_{i=1}^{n} \ell_i^2(x) - \sum_{i=1}^{n} \frac{\partial \ell_i}{\partial x_i}(x).$$

Then the solution $u(t,x)$ *for the Duncan-Mortensen-Zakai equation* (3) *is reduced to the solution* $\widetilde{u}(t,x)$ *for the Kolmogorov equation*

$$\frac{\partial \widetilde{u}}{\partial t}(t,x) = \frac{1}{2}\Delta\widetilde{(t,x)} - \sum_{i=1}^{n}\left(\ell_i(x) + \frac{\partial G}{\partial x_i}(x)\right)\frac{\partial \widetilde{u}}{\partial x_i}(t,x) - \sum_{i=1}^{n}\left(\frac{\partial \ell_i}{\partial x_i}(x) + \frac{\partial^2 G}{\partial x_i^2}(x)\right)\widetilde{u}(t,x)$$

where

$$\widetilde{u}(t,x) = e^{c(t)+G(x)+\sum_{i=1}^{n} a_i(t)x_i - F(x+b(t))}u(t,x+b(x))$$

and $a_i(t)$, $b_i(t)$, *and* $c(t)$ *satisfy the following system of* ODEs*: For* $1 \le i \le n$

$$b_i'(t) - a_i(t) - \sum_{j=1}^{n} d_{ij}b_j(t) + \sum_{j=1}^{m} c_{ji}y_j(t) = 0 \tag{4}$$

$$a_i'(t) + \sum_{j=1}^{n} d_{ji}b_j'(t) - \frac{1}{2}\sum_{j=1}^{n}(\eta_{ij}+\eta_{ji})b_j(t) = 0 \tag{5}$$

$$\begin{aligned} c'(t) &+ \frac{1}{2}\sum_{i=1}^{n}(b_i'(t))^2 - \sum_{i=1}^{n} a_i(t)b_i'(t) - \frac{1}{2}\sum_{k,j=1}^{n}\eta_{ij}b_i(t)b_j(t) \\ &- \frac{1}{2}\sum_{i=1}^{n}\eta_i b_i(t) + \sum_{i=1}^{n} d_i b_i'(t) = 0. \end{aligned} \tag{6}$$

Theorem 2 *Consider the Yau filtering system* (1) *with arbitrary initial condition satisfying* (Y1), (Y2) *and* (Y3) *in* Theorem 1. *Then the solution* $u(t,x)$ *for the Duncan-Mortensen-Zakai equation* (3) *is reduced to the solution* $\widetilde{u}(t,x)$ *for the Kolmogorov equation*

$$\begin{aligned} \frac{\partial \widetilde{u}}{\partial t}(t,x) &= \frac{1}{2}\Delta\widetilde{u}(t,x) - \sum_{i=1}^{n}\ell_i(x)\frac{\partial \widetilde{u}}{\partial x_i}(t,x) \\ &+ \frac{1}{2}\left(\sum_{i=1}^{n}\ell_i^2(x) - \sum_{i=1}^{n}\frac{\partial \ell_i}{\partial x_i}(x) - \eta(x)\right)\widetilde{u}(t,x) \end{aligned}$$

where

$$\widetilde{u}(t,x) = e^{c(t)+\sum_{i=1}^{n} a_i(t)t_i - F(x+b(t))} u(t, x+b(t))$$

and $a_i(t)$, $b_i(t)$ *and* $c(t)$ *satisfy* ODEs (4), (5) *and* (6).

Theorem 3 *Consider the Yau filtering system* (1) *with arbitrary initial condition satisfying* (Y1), (Y2) *and* (Y3) *in* Theorem 1. *Then the solution* $u(t,x)$ *for the Duncan-Mortensen-Zakai equation* (3) *is reduced to the solution* $\widetilde{u}(t,x)$ *for the Kolmogorov equation*

$$\begin{aligned} \frac{\partial \widetilde{u}}{\partial t}(t,x) &= \frac{1}{2}\Delta\widetilde{u}(t,x) - \sum_{i=1}^{n} f_i(x)\frac{\partial \widetilde{u}}{\partial x_i}(t,x) \\ &+ \frac{1}{2}\left(\sum_{i=1}^{n} f_i^2(x) - \sum_{i=1}^{n}\frac{\partial f_i}{\partial x_i}(x) - \eta(x)\right)\widetilde{u}(t,x) \end{aligned}$$

where

$$\widetilde{u}(t,x) = e^{c(t)+\sum_{i=1}^{n} a_i(t)x_i + F(x) - F(x+b(t))} u(t, x+b(t))$$

and $a_i(t)$, $b_i(t)$ *and* $c(t)$ *satisfy* ODEs (4), (5) *and* (6).

Notice that the explicit recursive filter for exact filtering system was previously derived only for maximal rank case (i.e., rank of (c_{ij}) is n), see [Be] for a particular maximal rank case and [T-W-Y] for general maximal rank case. Our result is an important breakthrough for the last 20 years because our method works for most general filtering systems and we no longer need maximal rank condition to construct explicit recursive filter and we can allow arbitrary condition. The significance of our results are that this estimation problem have been factored into two parts: (1) the off-line calculation of the Kolmogorov type equation which does not depend on the observations and (2) the on-line solution of a finite linear system of ordinary differential equations, which can be realized in real time.

Yau has given several lectures in the Department of Electrical Engineering, National Cheng Kung University, Tainan, Taiwan. Under Yau's direction, Professor Yen-Tai Lai and his students has successfully implemented an ODEs solver for the Yau filtering system. This ODEs solver has three advantages: 1) the data processing is parallel-input parallel-output so that it is faster than other strategies of serial-input parallel-output; 2) the solving processes can be divided by fan-in algorithm that can speed up the whole performance of the ODE system in Yau filter, 3) the ODEs solver can be easily implemented. This ODEs solver not only can estimate the solution of the ODEs in the Yau filter, but also can solve the exponential problem required in Yau filter. The performance of the ODEs solver is fast enough for real applications.

References

[Be] Benés, V., Exact finite dimensional filters for certain diffusions with nonlinear drift, *Stochastics* **5** (1981), 65-92.

[Br] Brockett, R.W., Nonlinear systems and nonlinear estimation theory, in *The Mathematics of Filtering and Identification and Application,* M. Hazewinkel and J.S. Willems, eds., Reidel, Dordrecht, 1981.

[Br-Cl] Brockett, R.W. and Clark, J.M.C., The geometry of the conditional density functions, in *Analysis and Optimization of Stochastic Systems,* O.L.R. Jacobs, et al., eds., Academic Press, New York, 1980, 299-309.

[Ch] Chen, J., On Ubiquity of Yau Filters, Proceedings of the American Control Conference, Baltimore, Maryland, June (1994), 252-254.

[Ch-Ya] Chiou, W.L., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift II: Brockett's problem of classification of finite dimensional estimation algebras, *SIAM J. Control and Optimization* **32**, no. 1 (1994), 297-310.

[Ch-Ya1] Chen, J. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift VI: Linear structure of Ω matrix, *Mathematics of Control, Signals and Systems,* **9** (1996), 370-385.

[Ch-Ya2] Chen, J. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift VII: Mitter conjecture and structure of η (with Jie Chen), *SIAM J. Control and Optimization,* **35**, no. 4, July (1997), 1116-1131.

[C-L-Y1] Chen, J., Leung, C.W., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift IV: Classification of finite dimensional estimation algebras of maximal rank with state space dimension three, *SIAM J. Control and Optimization* **34**, no. 1, (1996), 179-198.

[C-L-Y2] Chen, J. Leung, C.W., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift VIII: Classification of finite dimensional estimation algebra of maximal rank with state space dimension 4, *SIAM J. Control and Optimization,* **35**, no. 4, July (1997), 1132-1141.

[D-T-W-Y] Dong, R.T., Tam, W.S., Wong,W.S., and Yau,S.S.-T., Structure and classification theorems of finite dimensional exact estimation algebras, *SIAM J. Control and Optimization,* **29** (1991), 866-877.

[D-W-Y] Dong, R.T., Wong, W.S., and Yau, S.S.-T., Filtering system with finite dimensional estimation algebras, *IEEE Automatic Control,* **42** (11), (1997), 1601-1606.

[Ha] Hazewinkel, M., Lectures on linear and nonlinear filtering, in *Analysis and Estimation of Stochastic Mechanical Systems,* CISM Courses and Lectures 303, W. Shiehlen and W. Wedig, eds., Springer, Vienna, 1988, 103-135.

[Ha-Pa] Haussmann, U.G., and Pardouxx, E., A conditionally almost linear filtering problem with non-Gaussian initial condition, *Stochastics* **23** (1988), 241-275.

[Hu-Ya1] Hu, G.Q. and Yau, S.S.-T., Explicit solution of DMZ equation, Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, Dec. (1997), 4455-4459.

[Hu-Ya2] Hu, G.Q. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift XII: Linear and constant structure of Ω, (preprint).

[Hu-Ya3] Hu, G.Q. and Yau,S.S.-T., Finite dimensional filters with nonlinear drift XIII: Classification of finite dimensional estimation algebras of maximal rank with state space dimension less than or equal to five, (preprint).

[L-C-Y] Lai, Y.-T., Chen, P.-H. and Yau, M.-Y., An ODE Solver for the Yau Filtering System, (preprint)

[L-Y-Y] Liang, Z.G., Yau, S.-T., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift V: Solution to Kolmogorov equation arising from linear filtering with non-Gaussian initial condition, *IEEE Transactions on Aerospace and Electronic Systems,* **33**, no. 4, October (1997), 1295-1308.

[Mil] Mitter, S.K., On the analogy between mathematical problems of nonlinear filtering and quantum physics, *Ricerche di automatica* **10**, no. 2 (1979), 163-216.

[Mi2] Mitter, S.K., Lectures on nonlinear filtering and stochastic control, in *Nonlinear Filtering and Stochastic Control,* eds. S.K. Mitter and A. Moro, Springer Lecture Notes in Mathematics #972 (1983), 170-207.

[Ra-Ya] Rasoulian, A. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift IX: Construction of finite dimensional estimation algebra of non-maximal rank, Systems Control Letters, **30** (1997), 109-118.

[T-W-Y] Tam, L.F., Wong, W.S., and Yau, S.S.-T., On a necessary and sufficient condition for finite dimensionality of estimation algebras, *SIAM J. Control and Optimization,* **28**, no. 1 (1990), 173-185.

[Wo1] Wong, W.S., On a new class of finite dimensional estimation algebras, *Systems and Control Letters* **9** (1987), 79-83.

[Wo2] Wong, W.S., Theorems on the structure of finite dimensional estimation algebras, *Systems and Control Letters* **9** (1987), 117-124.

[Wo-Ya] Wong, W.S. and Yau, S.S.-T., The estimation algebra of nonlinear filtering systems, in *Mathematical Control Theory,* J. Baillieul and J.C.W. Willems, eds., Springer-Verlag, (1998), 33-65.

[Ya1] Yau, S.S.-T., Recent results on nonlinear filtering: New class of finite dimensional filters, Proceedings of the 29th Conference on Decision and Control at Honolulu, Hawaii, Dec. (1990), 231-233.

[Ya2] Yau, S.S.-T., Finite dimensional filters with nonlinear drifts I: A class of filters containing both Kalman filters and Benés filters, *J. Math. Systems, Estimation and Control* **4**, no. 2 (1994), 181-203.

[Ya-Le] Yau, S.S.-T. and Leung, C.-W., Recent result on classification of finite dimensional maximal rank estimation algebras with state space dimension 3, Proceedings of the 31st Conference on Decision and Control, Tucon, Arizona, Dec. (1992), 2247-2250.

[Ya-Ya1] Yau, S.-T., and Yau, S.S.-T., Explicit solution to a Kolmogorov equation, *Applied Mathematics and Optimization,* an international journal (to appear).

[Ya-Ya2] Yau, S.S.-T. and Yau, S.-T., New direct method for Kalman-Bucy filtering system with arbitrary initial condition, Proceedings of the 33rd Conference on Decision and Control, Lake Buena Vista, Florida, Dec. (1994), 1221-1225.

[Ya-Ya3] Yau, S.-T., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift XI: Explicit solutions of the generalized Kolmogorov equation in the Brockett Mitter program, *Advances in Mathematics,* **140** (1998), 156-189.

C. List of All Publications and Technical Reports

1. Addendum to "Finite -Dimensional Filters with Nonlinear Drift": A Response to F. Daum's Comments, (with Shing Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, 34, No.2 (1998), 57-58.

2. The estimation algebra of nonlinear filtering systems (with W.S. Wong), Mathematical Control Theory, Special Volume Dedicated to 60th Birthday of Brockett (J. Baillieul and J.C. Willems eds.), Springer Verlag, (1998), 33-65.

3. Finite Dimensional Filters with Nonlinear Drift XI: Explicit Solution of the Generalized Kolmogorov Equation in the Brockett- Mitter Program (with Shing-Tung Yau), Advances in Mathematics, 140, (1998), 156-189.

4. Recent results on classification of finite dimensional maximal rank estimation algebras with state space dimension up to 5 (with Guo-Qing Hu), Proceedings of the 37th IEEE Conference on Decision and Control, Dec. 16-18, Tampa, FL, (1998), 1311-1312.

5. Recent advance in computing Kolmogorov equation arising from nonlinear filtering (with Xi Wu and Ke-Pao Lin), Proceedings of the 37th IEEE Conference on Decision and Control, Dec. 16-18, Tampa, FL, (1998), 2904-2905.

6. Existence and Uniqueness and Decay Estimates for the Time Dependent Parabolic Equation with Application to Duncan-Mortensen-Zakai equation (with S.T.Yau), Asian Journal of Mathematics, Vol. 2 (1998), 1079-1149.

7. Lie algebraic method in nonlinear filtering with state space dimension up to 6 (with Larn-Ying Yeh), the 14th world congress of IFAC, Beijing, China, (1999), 97-102.

8. Approximate nonlinear output regulation based on the universal approximation theorem (with Jin Wang and Jie Huang), Proceedings of the World Multiconference on Systems, Cybernetics and Informations, Orlando, Florida, July 1999, Vol. 7, 218-225.

9. Wavelet representations of general signals, (with T. Bielecki, J. Chen and E. Lin), Nonlinear Analysis, Vol. 35, no. 1, (1999), 125-141.

10. Hessian Matrix Non-decomposition Theorem (with X. Wu and W.S. Wong), Mathematical Research Letter 6, (1999), 1-11.

11. Classification of Four-Dimensional Estimation Algebras (with Amid Rasoulian), IEEE Transactions on Automatic Control, Vol. 44, No. 4 (1999), 2312-2318.

12. Explicit Solution to Generalized Time-Varying Kolmogorov Equation (with Xi Wu), Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix, Arizona, December, (1999), 1897-1902.

13. Lectures on Systems, Control and Information (with Lei Guo), AMS/IP, Studies in Advanced Mathematics, 17 (2000).

14. Approximate nonlinear output regulation based on the universal approximation theorem (with Jin Wang and Jie Huang), International Journal of Robust and Nonlinear Control, 10, April 2000, 439-456.

15. Brockett's Problem on Nonlinear Filtering Theory, in Lectures on Systems, Control and Information, AMS/IP, Studies in Advanced Mathematics, 17 (2000), 177-212.

16. Real time solution of nonlinear filtering problem without memory I (with S.-T. Yau), Mathematical Research Letters 7, (2000), 1-23.

17. Existence of Solutions to Time Dependent Parabolic Equations with Unbounded Coefficients: Application to Duncan-Mortensen-Zakai Equations (with S.T. Yau), Proceedings of the American Control Conference, Chicago, June, (2000), 794-798.

18. Linear Filtering System with Arbitrary Initial Conditions (with Xi Wu), Proceedings of the American Control Conference, Chicago, June, (2000), 785-789.

19. Solution to Brockett's problem on finite-dimensional estimation algebras of maximal rank in nonlinear filtering, Proceedings of the 38th IEEE Conference on Decision and Control, Sydney, Australia, Dec. 2000, 292-297.

20. A study of Tracking-Differentiator (with X.J. Wang and Jie Huang), Proceedings of the 38th IEEE Conference on Decision and Control, Sydney, Australia, Dec. 2000, 4783-4788.

21. New proof of the linear structure of Wong matrix in nonlinear filtering (with X. Wu, J. Chen and W.L. Chiou), Proceedings of 19th Chinese Control Conference, Hong Kong, Dec. 2000, 384-388.

22. Finite dimensional filters with nonlinear drift XIII: Classification of finite-dimensional estimation algebras of maximal rank with state space dimension less than or equal to five (with GuoQing Hu and Wen-Lin Chiou), Asian Journal of Math, Vol. 4, No. 4, (2000), 905-932.

23. Finite dimensional filters with nonlinear drift X: Explicit Solution of DMZ Equation (with Guo-Qing Hu), IEEE Transactions on Automatic Control, Vol. 46, No. 1, (2001), 142-148.

24. Real time numerical solution to Duncan-Mortensen-Zakai equation (with S.T. Yau), (to appear) Foundations of Computational Mathematics (eds. R.A. DeVore, A. Iserles & E. Suli), Cambridge University Press, Cambridge, 2001.

25. The Consistency Problem on Content-based Pictorial Description in Pictorial Database Systems (with Qing-Long Zhang and Shi-Kuo Chang), 26 pp. in ms., (to appear) Communication in Information and Systems.

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Graduate Students

Ph.D. Students:

Amid Rasoulian, Ph.D.	Xuejun Wang
Guo-Qing Hu, Ph.D.	Yi Wang
Xi Wu, Ph.D.	Ebenezer Armah
Ke-Pao Lin, Ph.D.	Junfeng Ding

Master Students:

Zhen Liu	Dan Li	Qiuping Li
Zuojun Lin	Yanwen Liu	Shengfeng Lu
Yusong Wang	Jingjin Yu	Ling Zhou
Yingxu Xiang	Xiaojin Liu	Yunkai Liu
Ping Tao	Xiao Wu	Xiwei Yang
Yue Zhao	Jiangsong Lin	Qian Zhang
Weidong Niu	Xuan Ye	Qin Zhou
Yuan Wei	Qing Chang	Jie Li
Ying Gao	Lina Wen	Fuxiong Yang
Peng He	Yuhua Chen	Ke Fan
Yu Han	Fengying Hou	Weiyi Lu
Yehong Lin	Chengzhi Shen	Jie Wang
Li Wang	Zhaoming Wang	Wei Yuan

Report of Inventions

Yau has given several lectures in the Department of Electrical Engineering, National Cheng Kung University, Tainan, Taiwan. Under Yau's direction, Professor Yen-Tai Lai and his students has successfully implemented an ODEs solver for the Yau filtering system. This ODEs solver has three advantages:

1) the data processing is parallel-input parallel-output so that it is faster than other strategies of serial-input parallel-output;

2) the solving processes can be divided by fan-in algorithm that can speed up the whole performance of the ODE system in Yau filter,

3) the ODEs solver can be easily implemented.

This ODEs solver not only can estimate the solution of the ODEs in the Yau filter, but also can solve the exponential problem required in Yau filter. The performance of the ODEs solver is fast enough for real applications.

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ODEs solver for the Yau filtering system.

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Curriculum Vitae

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Education

- Ph.D (1976), The state University of New York at Stony Brook
- M.A. (1974), The State University of New York at Stony Brook

Positions

- Professor,University of Illinois at Chicago (1984-)
- Director, Laboratory of Control and Information (1993-)
- Managing Editor, Journal of Algebraic Geometry (1991-)
- Editors-in-Chief, Communications in Information and Systems (2000-)
- Visiting Professor, Harvard University (Winter 1999)
- Visiting Professor, University of Pisa, Italy (Spring 1990)
- Visiting Professor, Johns Hopkins University (1989 - 90)
- University Scholar, University of Illinois at Chicago (1987 -90)
- Visiting Professor, Institute Mittag-Leffler, Sweden (Winter 1987)
- Visiting Professor, Yale University (1984-85)
- Vistiting Associate Professor, University of Southern California (1983-84)
- Member, The Institute for Advanced Study (1981-82)
- Visiting Research Mathematician, Princeton University (Spring 1981)
- Associate Professor, University of Illinois at Chicago (1980-84)
- Alfred P.Sloan Research Fellow (1980-82)
- Benjamin Pierce Assistant Professor, Harvard University (1977-80)

- Member, The Institute for Advanced Study (1976-77)

Prizes and Awards

- Alfred P.Sloan Research Fellowship (1980-82)
- University Scholar, University of Illinois at Chicago (1987-90)
- C.M. Cha Fellow from Hong Kong Baptist University May-July, 1995
- Guggenheim Fellowship, 2000-2001
- Biographical note in American Men And Women of Science
- Biographical note in Who's Who in the World
- Biographical note in Who's Who in Science and Engineering
- Biographical note in Who's Who in the Midwest
- Biographical note in Who's Who in American Education
- Biographical note in Who's Who among Asian Americans

Membership

- Senior Member of the Institute of Electrical and Electronic Engineers
- Member of Society for Industrial and Applied Mathematics
- Member of American Mathematical Society

Grants

- Army Research Office (1989-2002)
- National Science Foundation (1976-88, 1989-2002)
- National Science Foundation Special Year Grant (1987-88)
- Research Board, University of Illinois at Chicago (1987, 1984, 1983, 1981)
- The Clark and the Topier Fund, Harvard University (1977-80)

Honors

- Open Invitation to spend 1984-86 as a Research Professor at Sonderforschungsbereich "Theoretische Mathematic", University of Bonn, Germany
- Invited one-hour address at the AMS Meeting, Worcester, Massachusetts, April 1985

- Invited one-hour address at the Swedish Mathematical Society Meeting, Linkoping, January 1987
- Invited to visit one week and give a colloquium lecture at Aarhus University, Denmark, February 1987
- Invitation to spend a month at Fudan University, Shanghai, People's Republic of China to give a series of lectures, November and December 1987
- Invitation to spend one week at the Institute of Mathematics, Academia Sinica, Beijing, People's Republic of China to give several lectures, December 1987
- Invitation to spend eight weeks at the University of Pisa, Italy to give a series of lectures, January - February 1990
- Invitation to spend 9 days at Global Analysis Research Center, Seoul National University, Korea to give a series of lectures, February 1992
- Invitation to spend one month at Nanjing University, People's Republic of China to give a series of lectures, May 1993
- Invitation to spend one month at National Taiwan University, Republic of China to give a series of lectures, May 1994
- Invitation to spend two months at Hong Kong Baptist University to give a series of lectures, May-July, 1997
- Invitation to spend two months as Visiting Professor at the University of Science and Technology of Hong Kong to give a series of lectures, 1997.
- Invitation to give 5 two-hours Distinguished Lecture Series on Control Theory, Special Year on Control Theory, October 1997, Morning Star Institute, Academia Sinica, China.
- Invitation to spend two months as Visiting Professor at the Chinese University of Hong Kong to give a series of lectures, 1998.
- Invited 45 minutes speaker at International Chinese Congress of Mathematics, Beijing, China, 1999.

Ph.D Student Supervision

- Yung Yu (1988)
- Craig Seeley (1988)
- Yi-Jing Xu (1990)

- Wen-Lin Chiou (1991)
- Chi-Wah Leung (1993)
- Tan Jiang (1993)
- Li-Xing Jia (1994)
- Jie Chen (1994)
- Amid Rasoulian (1995)
- Hon-Wing Cheng (1996)
- Qing-Long Zhang (1996)
- Guo-Qing Hu (1997)
- Zhi-Gang Liang (1998)
- Ke-Pao Lin (1999)
- Xi Wu (2000)

Selected Professional Activities

- Editors-in-Chief and Founder, Communications in Information and Systems, 2000.
- Managing Editor and Founder, Journal of Algebraic Geometry, 1991 - present
- Organizer, Minisymposium on CR Geometry, The Chinese University of Hong Kong, May, 1999
- Chair of the Technical Session at the IEEE Conference of Decision and Control, "Nonlinear Filtering II", San Diego, CA 1997, Tampa, FL, 1998
- Organizer, International Workshop on CR Manifolds, The Chinese University of Hong Kong, May, 1997
- General Chairman, IEEE International Conference on Control and Information, The Chinese University of Hong Kong, June, 1995
- Coorganizer, International Conference on Singularities and Complex Geometry, Beijing, China, June 1994
- Coorganizer, Wavelets and Large-Scale Image Processing, Chicago, Oct. 1994
- Coorganizer, Wavelets and their applications in PDE, a minisymposium during SIAM Annual Meeting, San Diego, CA, July 25-29, 1994

- Organizer, Minisymposium on wavelets at the Third SIAM Conference on Linear Algebra in Signals, Systems and Control, University of Washington, Seattle, August,1993
- Organizer, Wavelets and its application at IEEE Regional Conference on Aerospace Control Systems, Rockwell Science Center, Thousand, CA, May 1993
- Co-organizer, Emerging Computational Advances in Systems and Control 31st IEEE Conference on Decision and Control, Tucson, Arizona, December 1992
- Organizer, Midwest Algebraic Geometry Conference at the University of Illinois at Chicago, March 1988
- Organizer, National Science Foundation Special Year Algebraic Cycles Conference at the University of Illinois at Chicago, March 1988
- AMS Special Session Chairman on Singularities and Complex Geometry, Worcester, Massachusetts, April 1985
- AMS Special Session Chairman on Differential Geometry of Submanifolds, Worcester, Massachusetts, April 1985

List of Publications

1. Two theorems on higher dimensional singularities, Math. Ann. 231 (1977), 55 -59.
2. On almost minimally elliptic singularities, Bull. Amer. Math. Soc. 83 (1977), 362-364.
3. The signature of smoothing of higher dimensional singularities, Bull. Amer. Math. Soc. 83 (1977), 1313-1315.
4. Normal singularities of surfaces, Proceedings of Symposia in Pure Mathematics 32 (1978), 195-198.
5. The signature of Milnor Fibers and duality theorems for strongly pseudoconvex manifolds, Invent. Math. 46 (1978), 81-97.
6. Hypersurface weighted dual graphs of normal singularities of surfaces, Amer. J. Math. 101 (1979), 761-812.
7. Gorenstein singularities with geometric genus equal to two, Amer. J. Math. 101 (1979), 813-854.
8. On strongly elliptic singularities, Amer. J. Math. 101 (1979), 855-884.
9. Normal two-dimensional elliptic singularities, Trans. Amer. Math. Soc. 254 (1979), 117-134.
10. On maximally elliptic singularities, Trans. Amer. Math. Soc. 257 (1980), 269-329.

11. Index theory for the boundaries of complex analytic varieties, Proc. Nat. Acad. U.S.A. 77 (1980), 1248-1249.

12. Deformations and equitopological deformations of strongly pseudoconvex manifolds, Nagoya Math. J. 82 (1981), 113-192.

13. Kohn-Rossi cohomology and its application to the complex Plateau problem I, Ann. of Math. 113 (1981) 67-110.

14. Sheaf cohomology on 1-convex manifolds, Recent Developments in Several Complex Variables, Ann. of Math. Study. 100 (1981), 429-452.

15. Existence of L2-integrable holomorphic forms and lower estimates of T1V, Duke Math. J. 48 (1981), 537-547.

16. Criterion for biholomorphic equivalence of isolated hypersurface singularities, (with John Mather), Proc. Nat. Acad. Sci., U.S.A. 78 (1981), 5946-5947.

17. Milnor number and classification of isolated singularities of holomorphic maps, (with Bruce Bennett), Lecture Notes in Mathematics 949, Springer-Verlag (1982), 1-34.

18. s^{n-1} invariant for isolated n-dimensional singularities and its application to moduli problems, Amer. J. Math. 104 (1982), 829-841.

19. Classification of isolated hypersurface singularities by their moduli algebras, (with John Mather), Invent. Math. 69 (1982), 243-251.

20. Various numerical invariants for isolated singularities, Amer. J. Math. 104 (1982), 1063-1100.

21. On irregularity and geometric genus of isolated singularities, Proc. Symp. Pure Math. 40, Part 2 (1983), 653-662.

22. Milnor algebras and equivalent relations among holomorphic functions, Bull. Amer. Math. Soc. 9 (1983), 235-239.

23. Continuous family of finite dimensional representations of a solvable Lie algebra arising from singularities, Proc. Nat. Acad. Sci. U.S.A. 80 (1983), 7694-7696.

24. Criteria for right-left equivalence and right equivalence of holomorphic functions with isolated critical points, Complex Analysis Several Complex Variables, Proc. Symp. Pure Math. 41 (1984), 291-297.

25. Riemann-Roch theorem for strongly pseudoconvex manifolds of dimension three, (with Paul Yang), Several Complex Variables, Proc. of the 1981 Hangzhous Conf., Birkhauser, Boston, (1984), 257-267.

26. An estimate of the gap of the first two eigenvalues in the Schrodinger operator,(with I.M. Singer, Bun Wong and Shing-Tung Yau), Ann. Scuola Norm. Sup., Pisa, Classe di Scienze, Serie IV, Vol. XII, N.2 (1985), 319-333.

27. Solvable Lie algebras and generalized Cartan Matrix arising from isolated singularities, Math. Z. 191 (1986), 489-506.

28. Singularities defined by sl(2,C)invariant polynomials and solvability of Lie algebras arising from isolated singularities, Amer. J. Math. 108 (1986), 1215-1240.

29. Lie algebras and their representations arising from isolated singularities: Computer method in calculating the Lie algebra and their cohomology, (with Max Benson), Adv. Stud. Pure Math. 8, Complex Analytic Singularities (1986), 3-58.

30. A necessary and sufficient condition for a local commutative algebra to be a moduli algebra:weighted homogeneous case, Adv. Stud.Pure Math. 8, Complex Analytic Singularities (1986), 687-697.

31. Some surfaces covered by the ball and a problem in finite groups, (with G.D.Mostow), Lecture Notes in Math.Vol.1271, Springer-Verlag, Proc. of a Symposium in Honor of T.A, Springer, (1987), 201-228.

32. Holomorphic symmetry, (with Blaine Lawson), Ann. Sci. Êcole Norm. Sup. 4e series, t. 20 (1987), 557-577

33. Classification of Jacobian ideals invariant by sl(2,C) actions, Mem. Amer. Math. Soc. 72 (1988), 1-180.

34. Topological types and multiplicities of isolated quasi-homogeneous surface singularities, Bull. Amer. Soc. 19 (1988), 447-454.

35. The inequality $\mu \geq 12pg-4$ for weakly elliptic hypersurface singularities, (with Yi-Jing Xu), Contemp. Math. 90 (1989), 317-344.

36. Topological types of isolated hypersurface singularities, Contemp. Math.101 (1989), 303-321.

37. Classification of topological types of isolated quasi-homogeneous two-dimensional hypersurface singularities, (with Yi-Jing Xu), Mancripta Math. 64 (1989), 445-469.

38. Multiplicity of isolated two-dimensional hypersurface singularities:Zariski problem, Amer. J. Math. 111 (1989), 763-767.

39. Recent results on finite dimensional exact estimation algebra, (with L.F.Tam and W.S.Wong), Proceedings of the 28th Conf. on Decision and Control at Tampa, Florida, Dec. (1989), 2574-2575.

40. On a necessary and sufficient condition for finite dimensionality of estimation algebras, (with L.F. Tam and W.S. Wong), SIAM J. Control Optim. 28 (1990), 173-185.

41. Variations of complex structures and variation of Lie algebras, (with Craig Seeley), Invent. Math. 99 (1990), 545-565.

42. Equivalence between isolated hypersurface singularities, (with Max Benson), Math. Ann. 287 (1990), 107-134.

43. An obstruction for smoothing of Gorenstein surface singularities, (with A.Libgober), Comment. Math. Helv. 65 (1990), 413-433.

44. Recent results on nonlinear filtering: New class of finite dimensional filters, Proceedings of the 29th Conf. on Decision and Control at Honolulu, Hawaii, Dec. (1990), 231-233.

45. A remark on moduli of complex hypersurface, Amer. J. Math., 113 (1990), 287-292.

46. Classification of finite dimensional estimation algebras and Schrodinger equation, (with R.T. Dong, L.F. Tam and W.S. Wong), SIAM J. Control Optim., Vol 29, No.4 (1991), 886-877.

47. Obstructions to embedding of real compact (2n-1)-dimensional CR-manifold in C^{n+1}, (with H.S. Luk), Proceedings of Symposia in Pure Mathematics Vol. 52 (1991), Part 3, 261-276.

48. Regularity to Harvey-Lawson's solution to complex Plateau problem, J. Differential Geom., 34 (1991) 425-429.

49. Solvability of the Lie algebras arising from singularities and non-isolatedness of the singularities defined by the invariant polynomials of sl(2,C), Amer. J. Math. 113 (1991), 773-778.

50. Algebraic methods in the study of simple-elliptic singularities, (with Craig Seeley), US-USSR Algebraic Geometry Symposium, Springer-Verlag, Lecture Notes in Mathematics 1479 (1991), 216-237.

51. Recent results on classification of finite dimensional estimation algebras: Dimension of state space ?, (with Wen-Lin Chiou), Proceedings of the 30th Conf. on Decision and Control, Brighton, England, Dec. 11-13 (1991), 2758-2760.

52. Topological types of seven classes of isolated singularities with C?-action, (with Yi-Jing Xu), Rocky Mountain J. Math.,Vol 22, (1992) 1147-1215.

53. Classification of gradient space as sl(2,C)-module I (with J. Sampson and Yung Yu), Amer. J. Math.114 (1992), 1147-1161.

54. Sharp estimate of number of integral points in tetrahedron, (with Yi-Jing Xu), Journal fur die reine und angewandte Mathematik 423 (1992), 199-219.

55. Recent result on classification of finite dimensional maximal rank estimation algebras with state space dimension3 (with Chi-Wah Leung), Proceedings of the 31st Conference on Decision and Control, Tucson, Arizona, Dec. (1992), 2247-2250.

56. Explicit fundamental solution to Kolmogorov equation(with S.T.Yau), Proceedings of the 31st Conference on Decision and Control, Tucson, Arizona, Dec. (1992), 1508-1511.

57. Classification of finite dimensional filters from Lie algebraic point of view, Transaction of the Ninth Army Conference on Applied Mathematics and Computing, (1992), 459-466.

58. Complex Hypersurface Singularities with Application in Complex Geometry, Algebraic Geometry and Lie Algebra, Lecture Notes Series Number 5, 1992, Research Institute of Mathematics, Global Analysis Research Center, Seoul National University, Seoul Korea.

59. Durfee conjecture and coordinate free characterization of homogeneous singularities, (with Yi-Jing Xu), Journal of Differential Geometry 37 (1993), 375-396.

60. Gorenstein quotient singularities in dimension three,(with Yung Yu), Mem. Amer. Math. Soc. Vol. 105 (1993), 1-88.

61. Topological and differentiable structures of the complement of an arrangement of hyperplanes, (with Tan Jiang) Proceedings of Symposia in Pure Mathematics, Vol.54 (1993), part 2, 337-357.

62. Topological invariance of intersection lattices of arrangements in CP2 (with Tan Jiang), Bulletin A.M.S. Vol.39, No.1 (1993) 88-93.

63. Cohomology and Splitting Criterion for holomorphic vector bundles on CPn (with Hing-Sun Luk) Math. Nachrichten 161 (1993), 223-238.

64. Finite dimensional estimation algebras of maximal rank with dimension of state space equal to 3 (with Jie Chen and Chi-Wah Leung), Tenth Army Conference on Applied Mathematics and Computing, (1993), 337-344.

65. Finite dimensional estimation algebras of maximal rank with dimension of state space equal to 4 (with Jie Chen and Chi-Wah Leung) European Control Conference, Groningen, The Netherland, June 28-July 1, (1993), 2126-2130.

66. Some remarks on wavelet transforms (with Tomasz Bielecki, Jie Chen, E. Bing Lin). IEEE Proceeding of the first Regional Conference on Aerospace Control Systems, May 25-27, (1993), 148-150.

67. Explicit construction of finite dimensional nonlinear filters with state space dimension 2, (with Wen-Lin Chiou), Proceedings of the 32nd Conference on Decision and Control, San Antonio, Texas, Dec. (1993), 710-713.

68. Classification of low dimensional estimation algebras(with Jie chen and Chi-wah Leung). Proceedings of 32nd Conference on Decision and Control, San Antonio, Texas, Dec. (1993), 732-734.

69. Wavelet and Wavelet Stieltjes transforms (with T. Bielecki, J. Chen and E. Lin) Proceedings of the 32nd Conference on Decision and Control, San Antonio, Texas, Dec. (1993), 3062-3063.

70. Finite dimensional filters with non-linear drifts I: A class of filters containing both Kalman filters and Benes filters, J. of Math. Systems, Estimation and Control., Vol. 4, (1994), pp.181-203.

71. Finite dimensional filters with nonlinear drift II: Brockett's problem on classification of finite dimensional estimation algebra (with Wen-Lin Chiou), SIAM J. Control and Optimization, vol.32, No.1 (1994) 297-310.

72. Diffeomorphic type of the complement of arrangement of hyperplanes, (with Tan Jiang) Composito Math, 92, (1994) 133-155.

73. Explicit formal solution to generalized Kolmogorov equation (with Shing Tung Yau), Eleventh Army Conference on Applied Mathematics and Computing, (1994), 373-386

74. Algebraic classification and obstructions of embedding of compact 3 dimensional CR manifolds in C^{n+1}. (with Hing Sun Luk and Yung Yu) , Math. Nachrichten 170 (1994), 183-200.

75. Computing the exponential of matrices, (with Hon Wing Cheng) Proceedings of the American Control Conference, Baltimore, Maryland. June, (1994), 3543-3547.

76. New direct mehtod for Kalman-Bucy filtering system with arbitrary initial condition, (with S.T. Yau) Proceedings of the 33rd IEEE conference on Decision and Control, Lake Buena Vista, Florida, Dec. 14-16,(1994), 1221-1225.

77. Random Wavelet Transformation and its Properties, SPIE Proceedings on Wavelet Applications in Signal and Image Processing II, Andrew Laine and Michael Unser, eds,Vol. 2303 July, (1994), 345-353 (with Tomasz R.Bielecki and Jie Chen).

78. Non-existence of negatively graded derivation of weighted homogeneous moduli algebra (with Hao Chen and Yi-Jing Xu), Journal of Algebra, 172, (1995), 243-254.

79. The wavelet application to Kolmogorov equation, (with Zhigang Liang) Proceeding of International Conference on Control and Information, (1995), 271-276.

80. A report on explicit formulas for exp(tA), (with Hon-wing Cheng) Proceeding of International Conference on Control and Information, (1995), 69-75.

81. Recent results on classification of 4-dimensional estimation algebras (with Amid Rasoulian) Proceeding of International Conference on Control and Information, (1995), 371-374.

82. Random wavelet transfrom, algebraic geometric coding and their applications in compression and de-noising of signals, (with Tomasz Bielecki; Man K. Kwong; Li M. Song) Proceeding of International Conference on Control and Information, (1995), 283-289.

83. Explicit construction of finite dimensional nonlinear filters with state space dimension 3, (with Jie Chen and Chi-Wah Leung) Proceedings of the 34th IEEE Conference on Decision and Control, New Orleans, Louisiana, Dec. 13-15, (1995), 4030-4034.

84. Construction of new finite dimensional nonlinear filters, (with Amid Rasoulian) Proceedings of the 34th IEEE Conference on Decision and Control, New Orleans, Louisiana, Dec. 13-15, (1995), 4002-4005.

85. Invariants of Strongly Pseudoconvec CR manifolds (with Hing-Sun Luk), Five Decades as a Mathematician and Educator, On the 80th Birthday of Professor Yung-Chow Wong, World Scientific, (1995), 279-305.

86. Invariant Kohn-Rossi cohomology and obstruction to embedding of compact real (2n-1)-dimensional CR manifolds in C^N, (with H.-S.Luk), Journal of the Mathematical Society of Japan, Vol. 48, (1996), 61-68.

87. Finite dimensional filters with nonlinear drift IV: classification of finite dimensional maximal rank estimation algebra with dimension of state space equal to 3, (with Jie Chen, Chi-Wah Leung), SIAM J. Control and Optimization, Vol. 34, No.1 (1996). 179-1 98.

88. Direct method without Riccati equation for Kalman-Bucy filtering system with arbitrary initial condition, (with Guo-Qing Hu), 13th World Congress IFAC, San Francisco, June 30-July 5, Volume H, (1996), 469-474.

89. Microlocal characterization of quasihomogeneous singularities (with Yijing Xu), Amer. J. Math, 118, (1996), 387-399.

90. Iconic indexing and maintenance of spatial relationships in image databases, (with Qing-Long Zhang and Shi-Kuo Chang) 12 pp. in ms. Conference on Multimedia Storage and Archiving Systems, SPIE's Photonics East 96 Symposium, Boston, MA, Nov. 18-22, 1996.

91. Explicit solution of a Kolmogorov equation (with S.T.Yau), Applied Mathematics and Optimization, An International Journal, 34, (1996), 231-266.

92. A unified approach to indexing images in image databases, (with Qing-Long Zhang and Shi-Kuo Chang), Proceedings of First International Workshop on Image Databases and Multi-Media Search, Amsterdam, The Netherland, August 22-23, 1996, 99-106.

93. A sharp estimate of the number of integral points in a 4-dimensimal tetrahedron (with Yi-Jing Xu), Journal fur die reine und ange wandte Mathematik, 473 (1996), 1-23.

94. The Structure of Ω-matrix in nonlinear filters (with Amid Rasoulian), Proceedings of the 35th Conference on Decision and Control, Kobe, Japan (1996), 1083-1087.

95. Parallel ODE-solvers for Kalman-Bucy Filter with Arbitrary Initial Condition (with Ho-Wing Cheng), Proceedings of the 35th Conference on Decision and Control, Kobe, Japan (1996), 4138-4145.

96. New direct method for Yau filtering system with arbitrary initial conditions (with Guo-Qing Hu), Proceedings of the 35th Conference on Decision and Control, Kobe, Japan (1996), 2539-2544.

97. A unified approach to iconic indexing, retrieval and maintenance of spatial relationships in image databases, (with Qing-Long Zhang and Shi-Kuo Chang), Journal of Visual Communication and Image Representation Vol. 7, No. 4 (Special Issue), Dec. (1996), 307-324.

98. Finite dimensional filters with nonlinear drift VI: Linear structure of Ω matrix,(with Jie Chen), Mathematics of Control, Signals and Systems, 9(1996), 370-385.

99. Complete Algebraic CR Invariants of codimension 3 strongly pseudoconvex CR manifolds (with Hing Sun Luk), Studies in Advanced Mathematics Vol. 5, Singularities and Complex Geometry, AMS/IP, (1997), 175-182.

100. Complement of arrangement of hyperplanes (with Tan Jiang), Studies in Advanced Mathematics Vol. 5, Singularities and Complex Geometry, AMS/IP, (1997), 93-104.

101. Finite dimensional filters with nonlinear drift III: Duncan-Mortenson-Zakai equation with arbitrary initial condition for Kalman-Bucy filtering system and Benes filtering system (with Shing Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 4, Oct. (1997), 1277-1294.

102. More explicit formulas for the matrix exponential (with Hon Wing Cheng), Linear Algebra and Application, vol 262, (1997), 131-163.

103. Finite dimensional filter with nonlinear drift V: Solution to Kolmogorov equation arising from linear filtering with non Gaussian initial condition (with Zhigang Liang, and Shing -Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 4, Oct. (1997), 1295-1308.

104. Finite dimensional filters with nonlinear drift VII: Mitter conjecture and structure of η (with Jie Chen), SIAM J. Control and Optimization, Vol. 35, No. 4, July (1997), 1116-1131.

105. Finite dimensional filters with nonlinear drift VIII: Classification of finite dimensional estimation algebra of maximal rank with state space dimension 4 (with Jie Chen and Chi-Wah Leung), SIAM J. Control and Optimization, Vol. 35, No. 4, July (1997), 1132-1141.

106. Finite dimensional filters with non-linear drift IX: construction of finite dimensional estimation algebra of non-maximal rank (with Amid Rasoulion), Systems Control Letters, 30(1997), 109-118.

107. Contribution to Munuera's Problem on the Main Conjecture of Geometric Hyperelliptic MDS Codes (with Hao Chen), IEEE Trans. Inform. Theory, Vol. 43, No. 4, (1997), 1349-1354.

108. A new iconic indexing for 2D and 3D scenes (with Qing-Long Zhang), Proceeding of the Second Chinese World Congress on Intelligent Control and Intelligent Automation (CWCICIA 97), Xian, June 23-27, 1997, 1667-1672.

109. Explicit solution of DMZ equation (with Guo-Qing Hu), Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, Dec.(1997), 4455-4459.

110. Explicit construction of finite-dimensional nonlinear filters with state space dimension 4 (with Chi-Wah Leung, Jie Chen),Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, Dec.(1997), 4462-4466.

111. Filtering system with finite dimensional estimation algebras (with Rui-Tao Dong and Wing-Shing Wong), IEEE Automatic Control, 42(11), (1997), 1601-1606.

112. An Experimental Result in Image Indexing using GEP-2D Strings (with Qing-Long Zhang and Shi-Kuo Chang), Image Database and Multimedia Search (eds. A. Smeulders and R. Jain), World Scientific Publ. Co., 1997, 127-146.

113. Some remarks on compact strongly pseudoconvex CR manifolds (with H.S. Luk), Advanced Studies in Pure Mathematics 25, CR-Geometry and Overdetermind Systems (1997), 237-246.

114. Singularities and Complex Geometry (with Q.K. Lu and A. Libgobor), AMS/IP, Studies in Advanced Mathematics, 5 (1997).

115. Addendum to "Finite -Dimensional Filters with Nonolinear Drift": A Response to F. Daum's Comments, (with Shing Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, 34, No.2 (1998), 57-58.

116. Explicit computation of graph invariant for strongly pseudo-convex compact 3-dimensional rational CR manifolds (with Hing-Sun Luk), Compositio Mathematica, 114 (1998), 77-111.

117. The estimation algebra of nonlinear filtering systems (with W.S. Wong), Mathematical Control Theory, Special Volume Dedicated to 60th Birthday of Brockett (J. Baillieul and J.C. Willems eds.), Springer Verlag, (1998), 33-65.

118. Intersection lattices and the topological structures of complements of arrangements in CP^2 (with Tan Jiang), Ann. Scuola Norm. Sup. Pisa Cl. Sci (4), Vol. XXVI, (1998), 357-381.

119. Explicit Computation of Generalized Hamming Weights for Some Algebraic Geometric Codes (with Hao Chen and Hing Sun Luk), Advanced in Applied Math 21, (1998), 124-145.

120. Finite Dimensional Filters with Nonlinear Drift XI: Explicit Solution of the Generalized Kolmogorov Equation in the Brockett- Mitter Program (with Shing-Tung Yau), Advances in Mathematics, 140, (1998), 156-189.

121. Recent results on classification of finite dimensional maximal rank estimation algebras with state space dimension up to 5 (with Guo-Qing Hu), Proceedings of the 37th IEEE Conference on Decision and Control, Dec. 16-18, Tampa, FL, (1998), 1311-1312.

122. Recent advance in computing Kolmogorov equation arising from nonlinear filtering (with Xi Wu and Ke-Pao Lin), Proceedings of the 37th IEEE Conference on Decision and Control, Dec. 16-18, Tampa, FL, (1998), 2904-2905.

123. Counterexample to boundary regularity of strongly pseudoconvex CR submanifold: An addendum to the paper of Harvey-Lawson(with H.S. Luk), Annals of Math 148(1998), 1153-1154.

124. Existence and Uniqueness and Decay Estimates for the Time Dependent Parabolic Equation with Application to Duncan-Mortensen-Zakai equation (with S.T.Yau), Asian Journal of Mathematics, Vol. 2 (1998), 1079-1149.

125. Explicit Computation of Cohomological Algebra of Complement of Arrangement of Hyperplanes in C^3 and New Geometric Characterization of Supersolvable Arrangements (with Tan Jiang), Proceeding of International Conference on Algebra and Geometry, International Press, (1998), 47-73.

126. Lie algebraic method in nonlinear filtering with state space dimension up to 6 (with Larn-Ying Yeh), the 14th world congress of IFAC, Beijing, China, (1999), 97-102.

127. Approximate nonlinear output regulation based on the universal approximation theorem (with Jin Wang and Jie Huang), Proceedings of the World Multiconference on Systems, Cybernetics and Informations, Orlando, Florida, July 1999, Vol. 7, 218-225.

128. Wavelet representations of general signals, (with T. Bielecki, J. Chen and E. Lin), Nonlinear Analysis, Vol. 35, no. 1, (1999), 125-141.

129. Hessian Matrix Non-decomposition Theorem (with X. Wu and W.S. Wong), Mathematical Research Letter 6, (1999), 1-11.

130. Classification of Four-Dimensional Estimation Algebras (with Amid Rasoulian), IEEE Transactions on Automatic Control, Vol. 44, No. 4 (1999), 2312-2318.

131. Explicit Solution to Generalized Time-Varying Kolmogorov Equation (with Xi Wu), Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix, Arizona, December, (1999), 1897-1902.

132. Lectures on Systems, Control and Information (with Lei Guo), AMS/IP, Studies in Advanced Mathematics, 17 (2000).

133. Approximate nonlinear output regulation based on the universal approximation theorem (with Jin Wang and Jie Huang), International Journal of Robust and Nonlinear Control, 10, April 2000, 439-456.

134. Brockett's Problem on Nonlinear Filtering Theory, in Lectures on Systems, Control and Information, AMS/IP, Studies in Advanced Mathematics, 17 (2000), 177-212.

135. Real time solution of nonlinear filtering problem without meomry I (with S.-T. Yau), Mathematical Research Letters 7, (2000), 1-23.

136. Z_8-cyclic codes and quadratic residue codes (with Mei Hui Chiu and Yung Yu), Advanced in Applied Math. 25, (2000), 12-33.

137. Existence of Solutions to Time Dependent Parabolic Equations with Unbounded Coefficients: Application to Duncan-Mortensen-Zakai Equations (with S.T. Yau), Proceedings of the American Control Conference, Chicago, June, (2000), 794-798.

138. Linear Filtering System with Arbitrary Initial Conditions (with Xi Wu), Proceedings of the American Control Conference, Chicago, June, (2000), 785-789.

139. Solution to Brockett's problem on finite-dimensional estimation algebras of maximal rank in nonlinear filtering, Proceedings of the 38th IEEE Conference on Decision and Control, Sydney, Australia, Dec. 2000, 292-297.

140. A study of Tracking-Differentiator (with X.J. Wang and Jie Huang), Proceedings of the 38th IEEE Conference on Decision and Control, Sydney, Australia, Dec. 2000, 4783-4788.

141. New proof of the linear structure of Wong matrix in nonlinear filtering (with X. Wu, J. Chen and W.L. Chiou), Proceedings of 19th Chinese Control Conference, Hong Kong, Dec. 2000, 384-388.

142. Algebraic determination of isomorphism classes of the moduli algebras of $\tilde{E}_6$ singularities (with Hao Chen, and Craig Seeley), Math. Ann. 318, (2000), 637-666.

143. Finite dimensional filters with nonlinear drift XIII: Classification of finite-dimensional estimation algebras of maximal rank with state space dimension less than or equal to five (with GuoQing Hu and Wen-Lin Chiou), Asian Journal of Math, Vol. 4, No. 4, (2000), 905-932.

144. Finite dimensional filters with nonlinear drift X: Explicit Solution of DMZ Equation (with Guo-Qing Hu), IEEE Transactions on Automatic Control, Vol. 46, No. 1, (2001), 142-148.

145. Simple geometric characterization of supersolvable arrangements (with Tan Jiang and Larn-Ying Yeh), Rocky Mountain Journal of Mathematics, vol. 31, No. 1, (2001), 303-312.

146. Sharp polynomial upper estimate of number of positive integral points in Tetrahedron and coordinate free characterization of homogeneous polynomials (with Ke-Pao Lin), International Congress of Chinese Mathematicians, 1998, Beijing, China.

147. An example of real analytic strongly pseudoconvex hypersurfaces which is not holomorphically equivalent to any algebraic hypersurfaces, 17 pp. in ms (with Xiaojun Huang and Shanyu Ji), (to appear) Arkiv for Mathematik.

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149. Real time numerical solution to Duncan-Mortensen-Zakai equation (with S.T. Yau), (to appear) Foundations of Computational Mathematics (eds. R.A. DeVore, A. Iserles & E. Suli), Cambridge University Press, Cambridge, 2001.

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